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Thermoacoustic heating at the closed end of an oscillating gas column

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The problem of thermoacoustic heating at the closed end of a tube, in which small gas oscillations are maintained, leads in the case of adiabatic walls, and within the framework of a linear theory of the oscillations and a second-order theory of the heating effect, to singular behaviour of the equilibrium temperature at the tube end. Two cases are discussed: one with vanishing viscosity and one with viscosity tending to infinity. The singularities turn out to be similar in character and integrable in both cases.

1. Introduction

Heating effects at the closed end of oscillating gas columns can be spectacular, as in the case of the Hartmann–Sprenger (HS) tube (Sprenger 1954). In its original form, the HS-tube is driven by a supersonic jet; considering the complicated combination of flow and heat-conduction effects that occur, it is not surprising that the theoretical approach to the problem proves to be very difficult. HS-tubes driven by high subsonic and low supersonic Mach number jets were successfully constructed by Brocher; these proved to be accessible to a fluid mechanical analysis (Brocher & Maresca 1969; Brocher, Maresca & Bournay 1970), and later to a theory of the heating effects (Brocher & Maresca 1973; Brocher 1977). The main phenomena to be considered for the thermal theory are shock heating, viscous dissipation, and the mass and energy exchange between the penetrating jet and the gas in the tube.

A further step towards simplifying the problem was taken by Merkli & Thomann (1975), who considered small-amplitude acoustic oscillations in a tube, adequately described by linear theory. The heating effect is then of second order, and the 'thermoacoustic streaming' is described by a quadratic theory based on the linear solution, in analogy to Rayleigh's theory for the acoustic streaming of mass. Merkli & Thomann found experimentally – and in excellent agreement with theory – a thermoacoustic heating effect at the closed ends, and cooling at the antinode.

Merkli & Thomann considered an isothermal wall, but (depending on wall conductivity) the thermoacoustic heating causes an axial temperature stratification along the tube wall and in the gas. This fact, together with the investigation of thermoacoustic oscillations driven by externally imposed axial temperature gradients, have led the author (Rott 1975) to a theory of thermoacoustic streaming which includes the effect of an axial temperature gradient. A common mean axial temperature stratification is assumed for the gas (at rest) and the wall; once this is given, the second-order heat exchange between the gas and the wall can be calculated from the acoustic solution. The essential effect is the transverse temperature gradient caused by the displacement of the stratified gas relative to the wall. For small displacements,

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a linear theory and a subsequent second-order calculation are appropriate. Clearly such an approach is not applicable for large displacements like those in an HS-tube.

Even for small oscillations, however, the problem of the closed end becomes rather complicated in the case in which a finite section at the end is assumed to be completely heat-insulated, i.e. adiabatic. This notion is widely used, but actually needs some explanations. When oscillations start from rest, heat is exchanged between the gas and the wall. This leads ultimately to the build-up of a steady-state mean temperature distribution $T_m(x)$ varying along the tube axis. Such a distribution is only defined when the properties of sources and sinks along the tube are specified. In particular, the temperature distribution in a heat-insulated section is defined by the condition that after the build-up phase, in the steady state, *no* heat is exchanged between the gas and the wall, a condition that leads to the determination of the common temperature stratification $T_m(x)$ of the gas and the wall. To consider this condition fulfilled for a *section* of a tube is tantamount to neglecting the axial steady Fourier heat conduction both in the gas and the wall. For the gas, thermoacoustic streaming in the presence of an axial temperature gradient is found to carry heat much more effectively than the Fourier conduction caused by the same gradient, a fact well known experimentally and confirmed theoretically (Rott 1975). The wall is assumed to have low conductivity, a condition that can be realized by proper design of the tube (e.g. as a stack of short well-insulated sections).

It will be shown that in the case in which a section adjacent to the closed end of a thermoacoustic oscillator is completely heat-insulated, the temperature distribution $T_m(x)$ must have a singularity (i.e. infinity) at the closed end. This result is obtained in the framework of the theory of thermoacoustic streaming based on small oscillations and all the attendant simplifying assumptions.

By way of introduction, a qualitative description of the build-up is given that leads to this singular behaviour. From the work of Merkli & Thomann (1975) it is known that a closed end of an isothermal tube filled with oscillating gas is always heated, i.e. heat flows from the gas to the wall.† The flux per unit tube length is proportional to the product of the acoustic pressure and velocity. Now suppose that the heat sink at the end section of the tube that maintains the isothermal state becomes saturated, so that the temperature rises in that section, both in the gas and the wall, thus approaching adiabatic conditions. Then the attendant heat flux into the gas is proportional to the temperature gradient in the gas and to the square of the acoustic velocity (Rott 1975). At the closed end, the acoustic velocity is zero and both types of flux vanish; however, the second kind vanishes with the square of the velocity and thus cannot compensate for the flux of the first kind in the vicinity of the closed end unless the temperature gradient becomes very large. The ultimate steady-state singularity is obtained by setting the total resultant heat flux equal to zero.

Calculations following this outline are presented later; first, however, a different way to the determination of the singularity is discussed, which is applicable in important special cases. It is based on an ingenious 'heuristic argument' set forth by Gifford & Longsworth (1966) in connexion with the explanation of their 'pulse-tube' effect. These considerations precede the experiments and theory by Merkli & Thomann (1975) and the theory of Rott (1975), but apparently no attempts were made to connect the two ideas, until in the work of Wheatley *et al.* (1983) notice was taken of both approaches. The analysis shown later is based on a suggestion by Wheatley (private communication).

To understand the idea of Gifford & Longsworth, one has to recall that the heat

† An explicit solution for isothermal walls in the limit of infinite viscosity but finite Prandtl number was given by Thomann (1976).

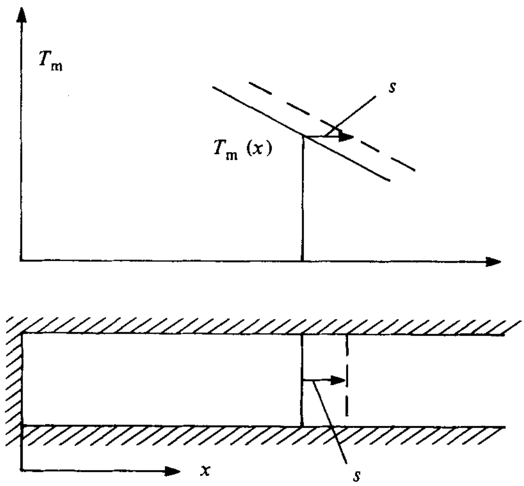

FIGURE 1. Heat-insulated end of a tube with a mean temperature distribution $T_m(x)$, in which gas oscillates (displacement s): ——, equilibrium distribution; ----, shifted distribution.

exchange between gas and wall due to oscillations is the consequence of the transverse temperature gradient caused by the displacement of the gas from the equilibrium position in which gas and wall have a common temperature stratification (figure 1). This, however, is not the only effect on the gas temperature to be considered: the displacement fluctuations in the gas are accompanied by temperature oscillations. In the core of the gas column not affected by friction and heat conduction, the connexion is isentropic. Near the closed end the effect of inertia is negligible, and the relation between the acoustic pressure p_a and the displacement s is given by the quasi-steady adiabatic relation between volume and pressure. This assumption is valid near the closed end over a tube section for which p_a can be taken as independent of x. The relation $(p_m + p_a)\, V^\gamma = \text{const}$ (where p_m is the mean pressure, uniform over the whole tube length) is independent of the gas temperature in the volume V and is even valid – as shown by a moment's consideration – for a temperature distribution $T_m(x)$ in V. Thus the following linearized relation holds (with a constant cross-section over the length considered):

$$\frac{p_a}{p_m} = -\gamma \frac{s}{x}. \tag{1.1}$$

But as

$$\frac{p_a}{p_m} = \frac{\gamma}{\gamma-1} \frac{T_a(x)}{T_m(x)}, \tag{1.2}$$

this means that

$$\frac{T_a(x)}{T_m(x)} = -(\gamma-1)\frac{s}{x}. \tag{1.3}$$

Now if

$$T_a(x) = \frac{dT_m(x)}{dx} s \tag{1.4}$$

the temperature oscillations exactly cancel the effect of the displacement of the stratified gas, and there is no transverse temperature gradient; (1.3) and (1.4) give

$$\frac{d \log T_m}{d \log x} = -(\gamma-1), \tag{1.5}$$

or

$$\frac{T_m(x)}{T_{m\,ref}} = \left(\frac{x}{x_{ref}}\right)^{-(\gamma-1)}, \tag{1.6}$$

showing the singularity of $T_m(x)$. From (1.2) it is seen that T_a is also singular, while p_a remains finite.

The fact that viscous boundary layers will modify this simple result was already anticipated by Gifford & Longsworth. In their paper, Wheatley *et al.* (1983) consider the 'heuristic argument' to be valid only for zero viscosity (vanishing Prandtl number σ). Even in this case, it does not appear to be obvious that the result (1.6) is independent of the thickness of the thermal boundary layer. However, following Wheatley (private communication), this fact can be proved; moreover, the solution (1.6) can be extended for a full tube length to a solution that is exact, albeit for a singular temperature distribution.

A different type of limiting process, presented by Müller (1982) in his dissertation, has led to another group of singular solutions. Müller argues that whenever one considers the limit $T_m \to \infty$, it is only consistent to assume that the viscosity and heat conduction of the gas also tend to infinity. According to kinetic theory, the viscosity μ of simple gases can be well represented (piecewise) by a power law $\mu \sim T_m^\beta$ with $\beta > 0.5$. Thus Müller assumes that at the closed end, dissipative effects dominate in the gas over the whole cross-section. Once this assumption is made, the value of β does not enter the results any further. What is more surprising is the fact that the Prandtl number drops out from Müller's formulas in the limiting analysis. As his result includes the case of zero Prandtl number, and the Gifford–Longsworth distribution is valid for any thickness of the thermal boundary layer, there is an apparent overlap in the regions of validity for the two theories. However, the Gifford–Longsworth result holds for $\mu = 0$, and Müller's for $\mu \to \infty$, with arbitrary thermal conductivity in both cases. In the first case the Prandtl number σ must be zero from the beginning; in the second, σ is arbitrary. The subsequent analysis shows that the order of steps by which the limit $\sigma = 0$ is reached is essential. Nevertheless, the difference in the results given by two theories is not overwhelming; Müller (1982) finds that

$$\frac{\mathrm{d}\log T_m}{\mathrm{d}\log x} = -\frac{\gamma-1}{1+c_j\gamma}, \quad c_j = \frac{3(1+j)}{2(7+j)}, \tag{1.7}$$

where $j = 0$ for two-dimensional ducts and $j = 1$ for round tubes.

The consequence to be drawn is that, once the singular behaviour at the closed end is admitted, the 'heuristic argument' is a strong one, and the investigation of its relation to thermoacoustic theory is justifiable.

2. Basic equations

Thermoacoustic oscillations of a gas column with axial temperature stratification are calculated under the simplifying assumption that the acoustic pressure $p_a = p e^{i\omega t}$ is constant in a cross-section, i.e. it depends only on the axial coordinate x. The acoustic velocity and temperature distributions are determined, however, with friction and heat conduction fully included in the radial direction, and neglected in the axial direction. Dynamically, the simplifications are the same as in boundary layer theory, but the dissipative layer does not have to be thin compared with the tube (or channel) radius.

Under these conditions, it has been found (Rott 1969) that p satisfies the equation

$$[1+(\gamma-1)f_j^*]p + \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{a^2}{\omega^2}(1-f_j)\frac{\mathrm{d}p}{\mathrm{d}x}\right] - \frac{a^2}{\omega^2}\frac{f_j^*-f_j}{1-\sigma}\theta\frac{\mathrm{d}p}{\mathrm{d}x} = 0, \tag{2.1}$$

where a is the speed of sound and $\theta = \mathrm{d} \log a^2/\mathrm{d}x = \mathrm{d} \log T_m/\mathrm{d}x$; f_j is a complex function of the variable

$$\eta_w = r_w \left(\frac{\mathrm{i}\omega}{\nu}\right)^{\frac{1}{2}}, \tag{2.2}$$

which in turn depends, through the kinematic viscosity, on the mean temperature T_m; with $\mu \sim T_m^\beta$ (say) and $\rho_m \sim 1/T_m$ (as $p_m = \text{const}$) one has

$$\nu \sim T_m^{1+\beta}, \quad \eta_w \sim T_m^{-\frac{1}{2}(1+\beta)}. \tag{2.3}$$

The functions f_j have been found to be given by

$$f_0 = \frac{1}{\eta_w} \tanh \eta_w \tag{2.4}$$

for channels ($j = 0$) with half-width r_w, and by

$$f_1 = \frac{2J_1(\mathrm{i}\eta_w)}{\mathrm{i}\eta_w J_0(\mathrm{i}\eta_w)} \tag{2.5}$$

for round tubes ($j = 1$) with radius r_w. Finally,

$$f_j^*(\eta_w) = f_j(\eta_w \sigma^{\frac{1}{2}}), \tag{2.6}$$

where σ is the Prandtl number.

Next, the quantity of main interest is the heat exchanged between wall and gas. Let $\bar{q}$ be the time-averaged heat flow per unit time and area (positive when directed into the wall). Then, from the energy equation of the gas it follows (Merkli & Thomann 1975; Rott 1975) that

$$2(\pi r_w)^j \bar{q} = -\frac{\mathrm{d}\bar{H}}{\mathrm{d}x}, \tag{2.7}$$

$$\bar{H} = 2\int_0^{r_w} \rho_m c_p \overline{T_1 u_1} (\pi r)^j \, \mathrm{d}r. \tag{2.8}$$

Here the index 1 for the first order quantities u, T, etc. indicates that the time-dependent (real) quantities are meant, and have to be time-averaged. (The corresponding quantities without index 1 are complex amplitudes.)

The quantity $\bar{H}$ could be called the axial heat-flux, as its rate of change with x gives $\bar{q}$. However, when a flexible piston is introduced at a position x, it extracts from the oscillation the power (positive for a piston driven from the left)

$$\mathscr{P} = 2\int_0^{r_w} \overline{p_1 u_1} (\pi r)^j \, \mathrm{d}r, \tag{2.9}$$

and there is a remainder of the average axial heat flux, to be called Q, so that

$$\bar{H} = \mathscr{P} + Q, \tag{2.10}$$

which gives

$$Q = \frac{2p_m}{R} \int_0^{r_w} \overline{s_1 u_1} (\pi r)^j \, \mathrm{d}r, \tag{2.11}$$

where s_1 is the entropy. What actually happens at a piston lies, in its details, outside the scope of the present theory. This could only be discussed (albeit with the use of considerable effort) with the help of the theory of Monkewitz (1979), which applies even in the case of strong axial changes. However, here the position is taken (supported by the investigations of Monkewitz) that the theory based on (2.1) is adequate up to the piston, in particular for the calculation of the power $\mathscr{P}$. Whether the heat Q is conducted through the piston or (by a modified field) in the tube near the piston cannot be decided in general, and for a particular model one would have to resort to the theory of Monkewitz.

The upshot of this discussion is that it is evidently more appropriate (following Wheatley *et al.* 1983) to call the quantity $\bar{H}$ the 'enthalpy flux' or 'total energy flux', instead of 'heat flux', which could be misleading at a piston.

It remains to determine $\bar{H}$, based on the known distributions of u and T (Rott 1969). The result is (Rott 1975)

$$\bar{H} = \left(\frac{\pi r_w}{2}\right)^j r_w \operatorname{Re}\left\{\left[u_e \tilde{p} - \frac{i}{\omega}\frac{\theta \rho_m a^2}{(\gamma-1)(1-\sigma)} u_e \tilde{u}_e\right] g_j\right\}, \tag{2.12}$$

where the tilde indicates the complex conjugate; the quantity

$$u_e = \frac{i}{\omega \rho_m}\frac{dp}{dx} \tag{2.13}$$

represents a measure of the acoustic velocity, and is equal to the external velocity for thin boundary layers; and the function g_j is given by

$$g_j = 1 - \frac{\sigma}{1+\sigma} f_j - \frac{1}{1+\sigma} \tilde{f}_j^*. \tag{2.14}$$

At a closed end $u_e = 0$ and $\bar{H} = 0$. Naturally, heat could be transferred through the endwall, but only by axial conduction, and it would be inconsistent to take this into consideration.

3. The Gifford–Longsworth solution

In the case when friction is neglected from the beginning, one has to set $f_j = 0$ in (2.1), while with $\sigma = 0$ the value of f_j^* remains finite:

$$\eta_w \sigma^{\frac{1}{2}} = r_w \left(\frac{i\omega\rho_m c_p}{k}\right)^{\frac{1}{2}},$$

where k is the heat conductivity. Equation (2.1) simplifies to

$$[1+(\gamma-1) f_j^*] p + \frac{d}{dx}\left(\frac{a^2}{\omega^2}\frac{dp}{dx}\right) - \frac{1}{\omega^2}\frac{da^2}{dx} f_j^* \frac{dp}{dx} = 0. \tag{3.1}$$

Following Wheatley (personal communication) we notice that if the sum of the terms multiplying f_j^* vanishes, i.e. if

$$(\gamma-1) p - \frac{1}{\omega^2}\frac{da^2}{dx}\frac{dp}{dx} = 0, \tag{3.2}$$

then the inner bracket in (2.12) (with u_e inserted from (2.13)) also vanishes for $\sigma = 0$, so that $\bar{H} = 0$ everywhere; simultaneously, one has to fulfill the equation

$$p + \frac{d}{dx}\left(\frac{a^2}{\omega^2}\frac{dp}{dx}\right) = 0, \tag{3.3}$$

which is the acoustic equation without friction and heat conduction.

The two equations (3.2) and (3.3) determine both p and that particular distribution of $a^2 \sim T_m$ for which $\bar{H} = 0$. Eliminating p from (3.2) and (3.3), one finds

$$\gamma \frac{da^2}{dx}\frac{dp}{dx} + (\gamma-1) a^2 \frac{d^2 p}{dx^2} = 0, \tag{3.4}$$

which can be integrated to give

$$\frac{dp}{dx} = Ca^{-2\gamma/(\gamma-1)}. \tag{3.5}$$

With the notation

$$\left(\frac{a}{a_0}\right)^{-2/(\gamma-1)} = z, \quad \frac{\omega x}{a_0} = \xi, \tag{3.6}$$

the equations (3.2)–(3.6) can be brought into the form

$$p = \frac{p_0}{c_0}\frac{dz}{d\xi}, \quad \frac{dp}{d\xi} = -\frac{p_0}{c_0}z^{\gamma}, \qquad (3.7), (3.8)$$

where p_0 is the pressure at the closed end, and c_0 is to be determined by proper normalization. Elimination of p between (3.7) and (3.8) leads to

$$\frac{d^2z}{d\xi^2} + z^{\gamma} = 0. \tag{3.9}$$

Multiplication by $dz/d\xi$ and integration gives

$$\left(\frac{dz}{d\xi}\right)^2 + \frac{2}{\gamma+1}z^{\gamma+1} = c_0^2 = \frac{2}{\gamma+1}, \tag{3.10}$$

whereby c_0 has been fixed for later convenience.

Further evaluation of z and p as functions of ξ is straightforward and leads to an exact solution. Its most important properties can be determined from the results thus far, considering the boundary conditions. At the closed end we have

$$z = 0, \quad \frac{dz}{d\xi} = c_0 \quad (\xi = 0). \tag{3.11}$$

Suppose the value of $\xi = \xi_0$ for which $dz/d\xi$ vanishes has been found from integration of (3.10); this is the open end

$$\frac{dz}{d\xi} = 0, \quad z = 1 \quad (\xi = \xi_0). \tag{3.12}$$

Comparison with (3.6) identifies the value of a_0: it is the speed of sound at the open end, where $z = 1$. Near to the closed end one has $z = c_0\xi$, or

$$\frac{T_m}{T_{m0}} = \left[\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}}\frac{\omega x}{a_0}\right]^{-(\gamma-1)} \tag{3.13}$$

This is the Gifford–Longsworth distribution. Here, in addition, the normalization of the singularity at the closed end has been connected to the (given) values at the open end.

The solution has to be visualized as an isentropic oscillation in a tube open at one end, with viscosity neglected, and the effects of a possible heat conduction nullified by the avoidance of any transverse temperature gradient which could be created by the axial displacement of the gas.

In the case in which the usual thermal boundary condition at the tube wall, which holds for infinite heat capacity of the tube, is replaced by a more general condition, (2.1) has to be replaced by a more complicated equation (Rott 1980). This leads to a change in (3.1) which causes the appearance of a factor multiplying f_j^* wherever it occurs. Thus the results of this section remain unaffected.

In 'reality', when the temperature at the closed end remains finite, heat-conduction losses at the closed end are unavoidable even at zero viscosity, and the 'open end' has to be replaced by a driving piston.

It also has to be noted that in the case in which $\bar{H}=0$ at a point (or a region) of a thermoacoustic oscillator, one cannot conclude that (3.2) hold there. $\bar{H}=0$ does not imply that a factor (in this case the square bracket in (2.12)) *must* be zero: the real part of the expression which gives $\bar{H}$ can vanish by a very different balance, as illustrated by the next example.

4. Müller's solution

The solution given by Müller (1982) for a closed end with $\bar{H}=0$ is valid for highly viscous flow in the limit $T_m \to \infty$, $\nu \to \infty$, $|\eta_w| \ll 1$. As a first step for the solution, an expansion of f_j, given by (2.4) and (2.5), is obtained in this limit. It is possible to join these expansions for $j=0$ and $j=1$ into one series by use of j as a parameter in the coefficients:

$$f_j = 1-\frac{1}{3+5j}\eta_w^2+\frac{2}{(3+5j)(5+7j)}\eta_w^4-\frac{17+5j}{(3+5j)^2(5+7j)(7+j)}\eta_w^6+\dots \tag{4.1}$$

The detailed analysis has shown that for the solution of (2.1) only the first two terms of (4.1) are needed; in this approximation one has

$$1-f_j^* \approx \sigma(1-f_j). \tag{4.2}$$

By use of this relation the basic equation (2.1) is reduced to

$$[\gamma-(\gamma-1)\sigma(1-f_j)]p+\frac{a^2}{\omega^2}\frac{d}{dx}\left[(1-f_j)\frac{dp}{dx}\right]=0. \tag{4.3}$$

Actually, near to the closed end it suffices to replace the first term in (4.3) by γp_0, and one obtains

$$(1-f_j)\frac{dp}{dx}=-\gamma p_0\omega^2\int\frac{dx}{a^2},$$

or, by use of u_e defined by (2.13) and with (4.1),

$$u_e=-(3+5j)\frac{p_0}{p_m}\frac{\nu}{r_w^2}T_m\int\frac{dx}{T_m}. \tag{4.4}$$

It is interesting to note that (4.4) shows p_0 and u_e *in* phase, in contrast with the exactly out-of-phase relation in §2 (see (2.13)). The balance $\bar{H}=0$ is now produced, as already noted previously, in a quite different way; with the notation $g_j=g_{j\mathrm{re}}+\mathrm{i}g_{j\mathrm{im}}$, (2.12) can now be rewritten as follows:

$$\bar{H}=\left(\frac{\pi r_w}{2}\right)^j r_w\left\{u_e p_0 g_{j\mathrm{re}}+\frac{\theta\rho_m a^2}{\omega(\gamma-1)}u_e^2\frac{g_{j\mathrm{im}}}{1-\sigma}\right\}. \tag{4.5}$$

Now f_j is introduced in (2.14) to determine the leading term of the real and imaginary parts of g_j. The result is

$$g_{j\mathrm{re}}=\frac{2\sigma}{(3+5j)(5+7j)}\frac{r_w^4\omega^2}{\nu^2}, \tag{4.6}$$

$$\frac{g_{j\mathrm{im}}}{1-\sigma}=-\frac{\sigma(17+5j)}{(3+5j)^2(5+7j)(7+j)}\frac{r_w^6\omega^3}{\nu^3}. \tag{4.7}$$

The condition $\bar{H} = 0$ reduces, by use of (4.4)–(4.7), to the condition

$$T_m \int \frac{dx}{T_m} \left(\frac{\theta\gamma}{\gamma-1}\right)\left(\frac{17+5j}{14+2j}\right) = -1. \tag{4.8}$$

Let T_m be proportional to x^{-n}, so that

$$T_m \int \frac{dx}{T_m} = \frac{x}{n+1}, \quad \theta = -\frac{n}{x}.$$

When this is inserted in (4.8), the exponent n is obtained in agreement with (1.7).

The surprising similarity of the singularities obtained here and in §3 has already been noted in §1. An important limitation to the usefulness of all singular solutions is, however, the fact that even a very weak – but unavoidable – axial conduction in the solid wall profoundly modifies the solutions; this will be the subject of future investigations. Nevertheless, as the singularities are integrable, it is possible to obtain an estimate (probably an upper bound) of the total amount of heat that can be stored in the gas near the end of a heat-insulated tube.

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